

# Causal Learning

Reading Group, Summer 2023

# Outline

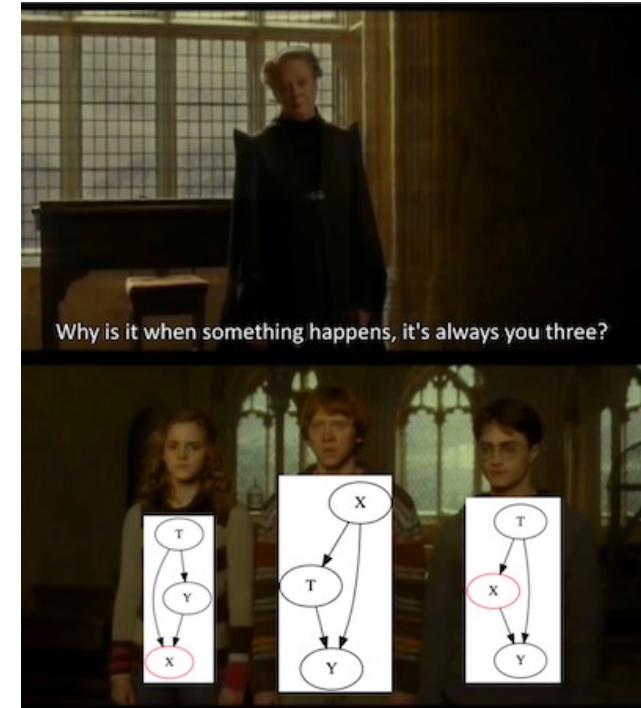
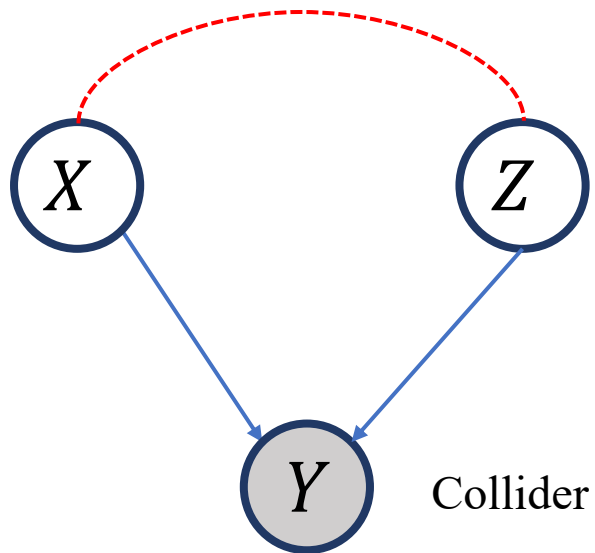
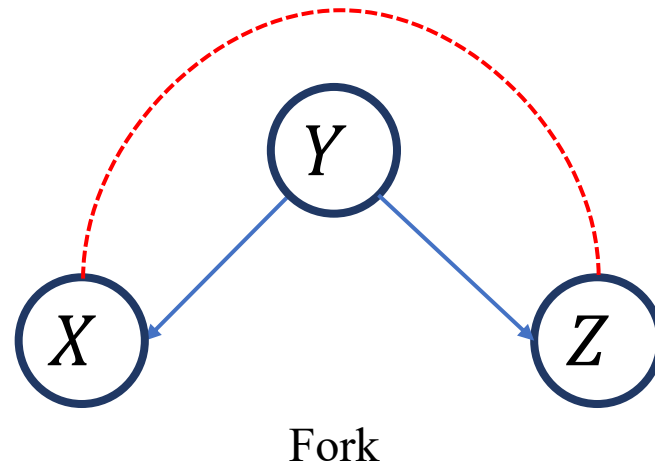
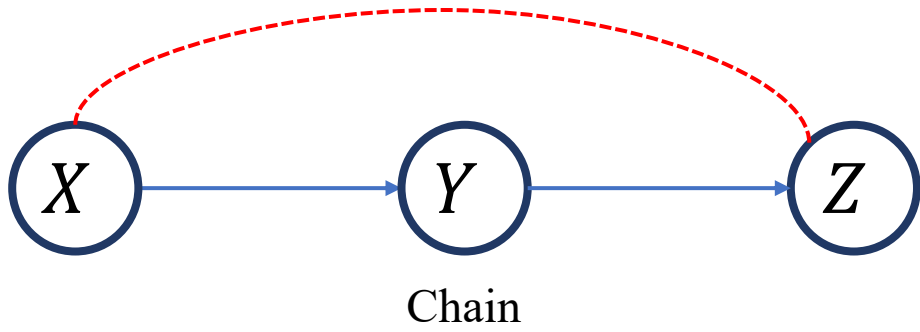
Blocked Paths and d-Separation

Backdoor Criterion

Frontdoor criterion

Do-Calculus

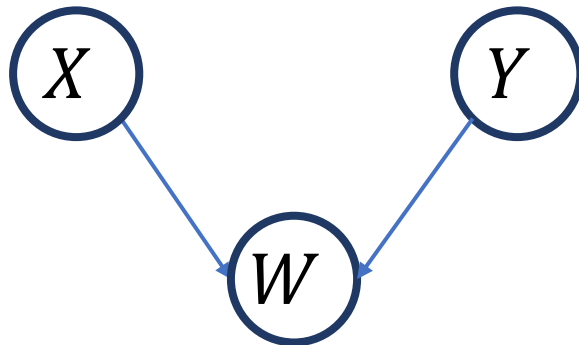
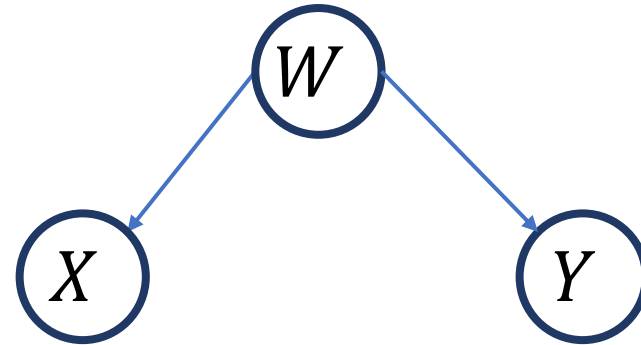
# Recap



# Blocked Paths

A path between nodes  $X$  and  $Y$  is blocked by a conditioning set  $Z$  if either of the following is true:

1. Along the path, there is a chain  $\dots \rightarrow W \rightarrow \dots$  or a fork  $\dots \leftarrow W \rightarrow \dots$  where  $W$  is conditioned on ( $W \in Z$ )
2. If there is a collider  $W$  on the path that is not conditioned on ( $W \notin Z$ ) and none of its descendants are condition on ( $de(W) \not\subseteq Z$ )

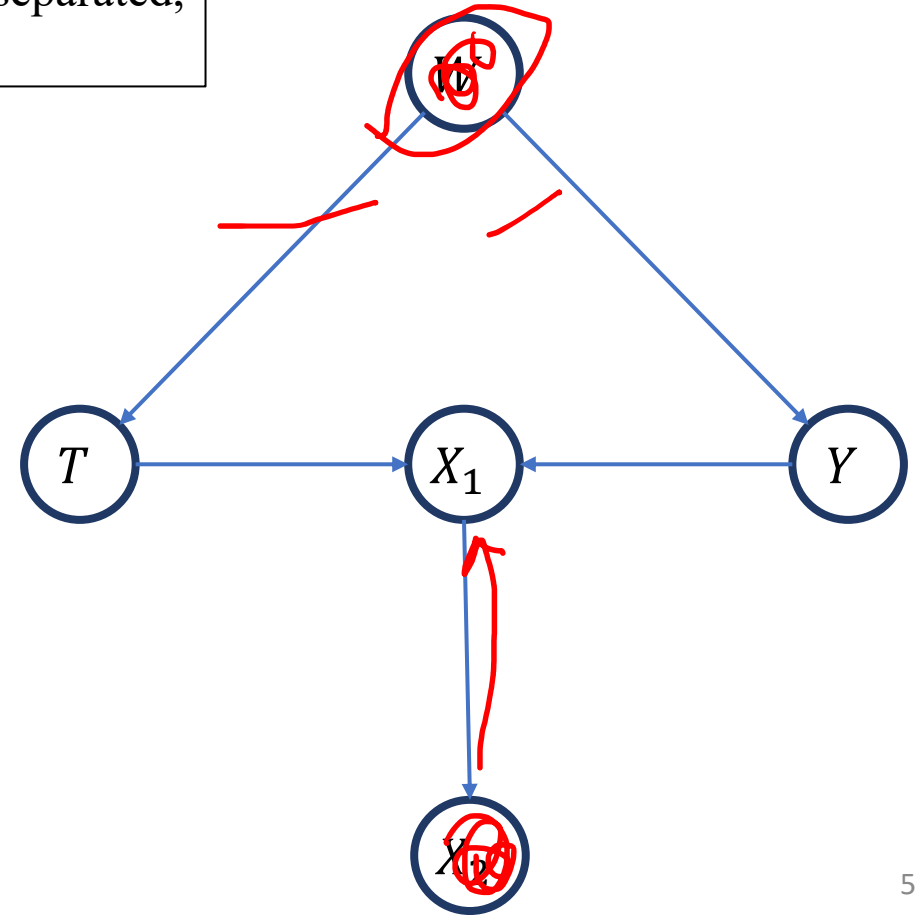
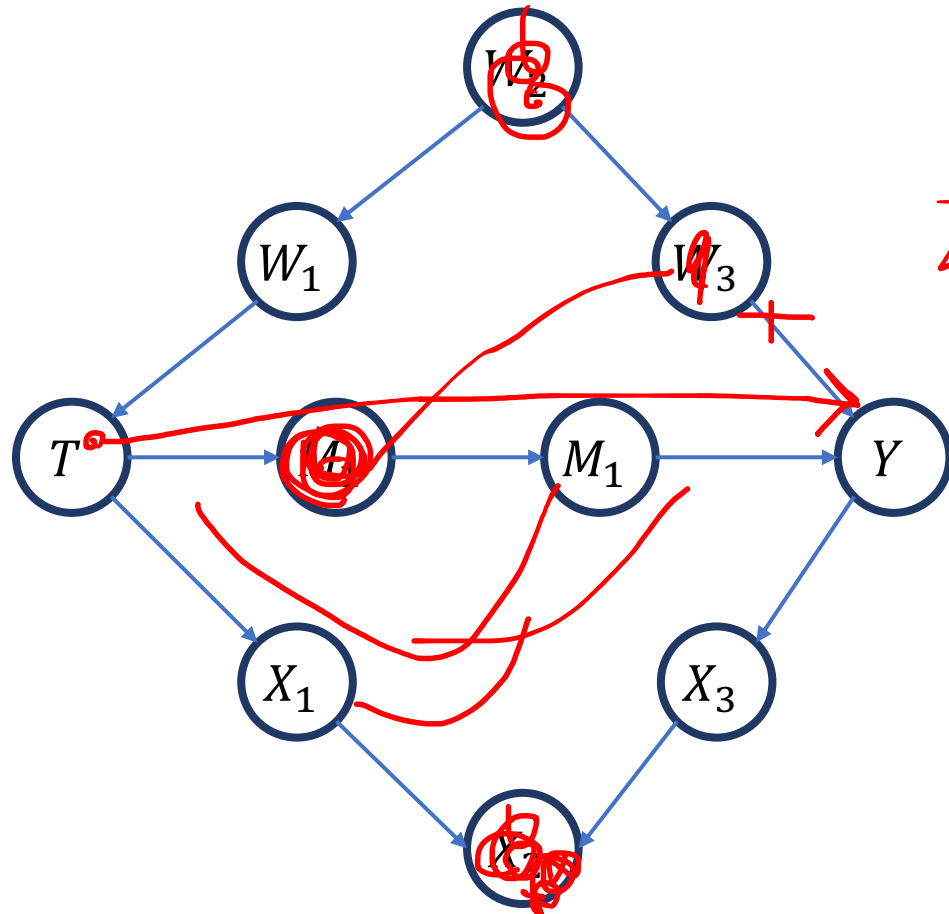


# d-Separation

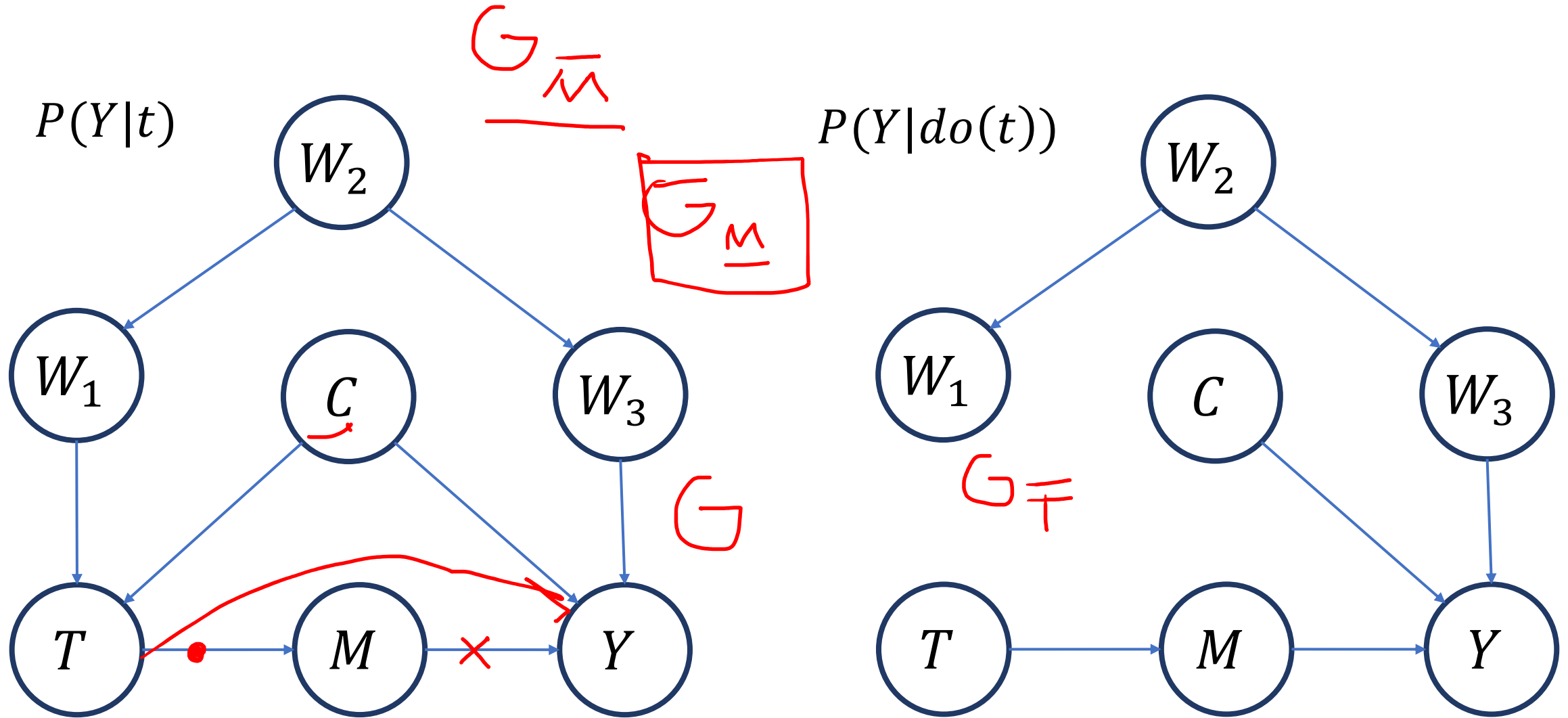
Two (sets of) nodes  $X$  and  $Y$  are d-separated by a set of nodes in  $Z$  if all of the paths between (any node in)  $X$  and (any node in)  $Y$  are blocked by  $Z$ .

Are  $T$  and  $Y$  d-separated,  
given....

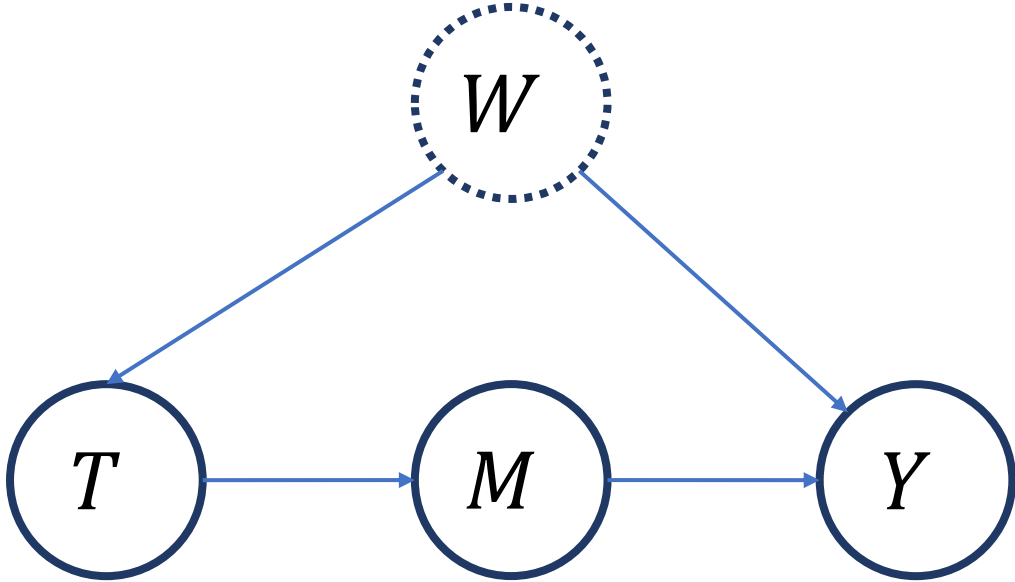
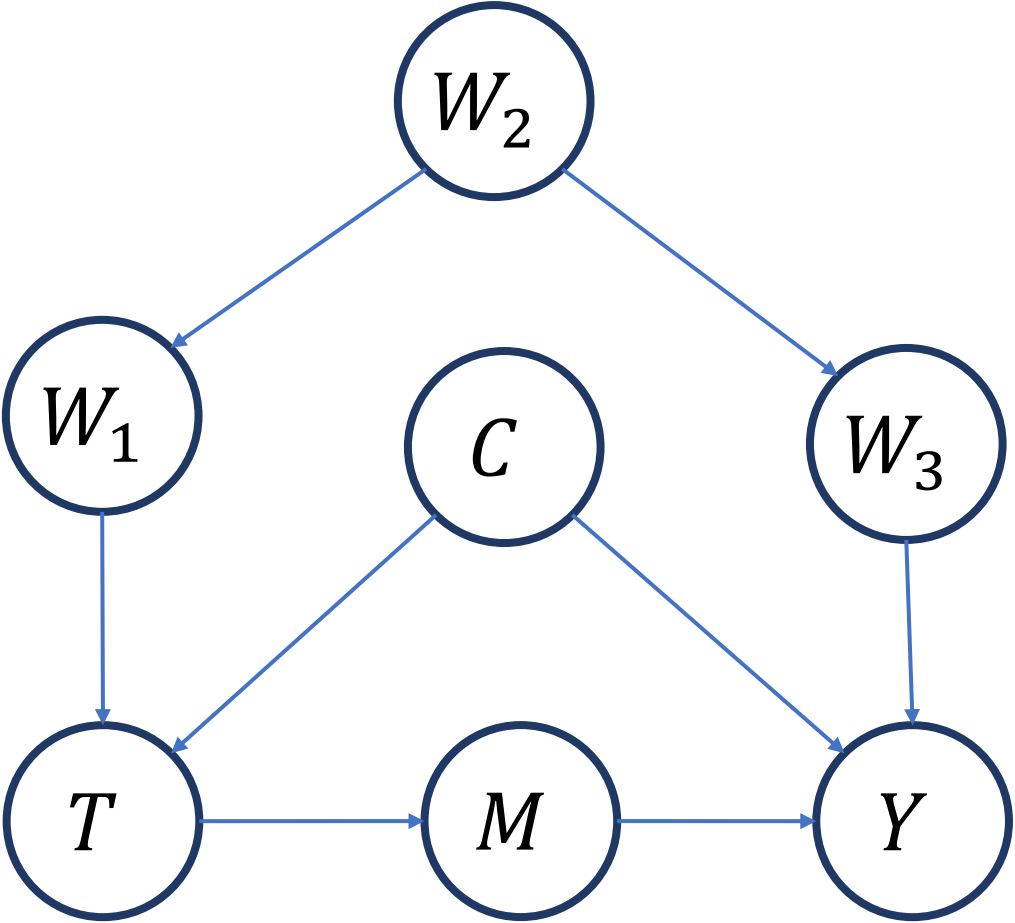
$Z = \{ \}$



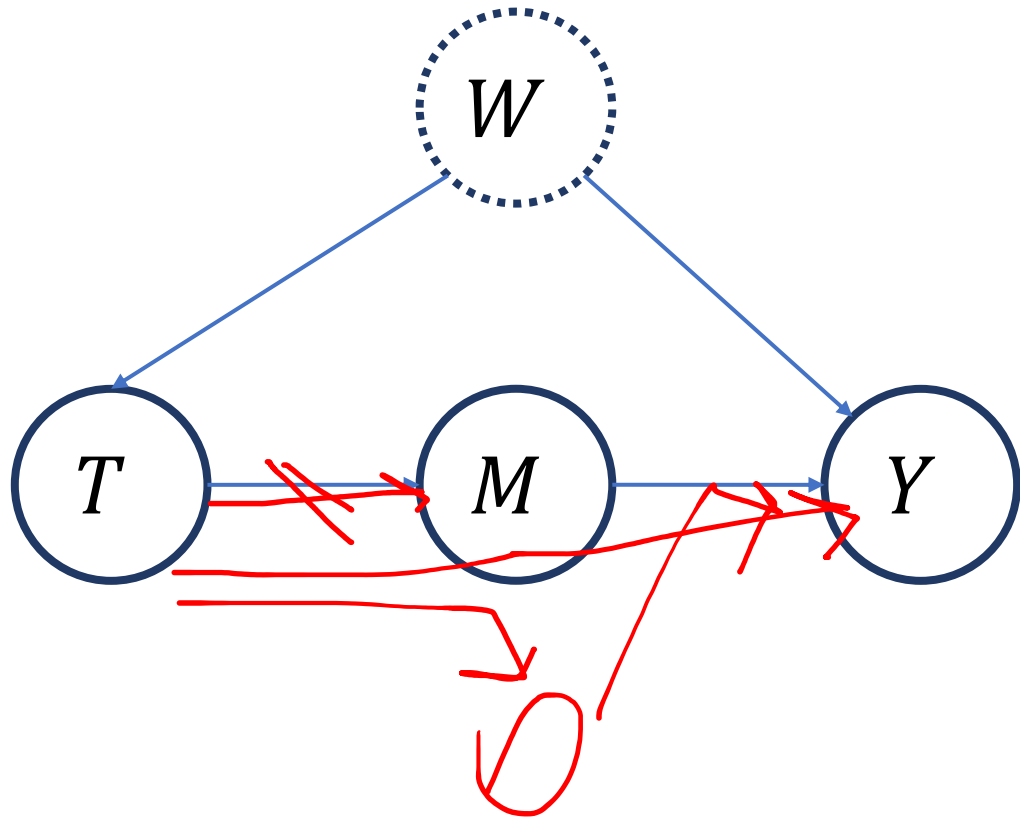
# Blocking backdoor paths



# Randomized Control Trial (RCT)



# Frontdoor Adjustment



1. Identify causal effect of  $T$  on  $M$
2. Identify causal effect of  $M$  on  $Y$
3. Combine

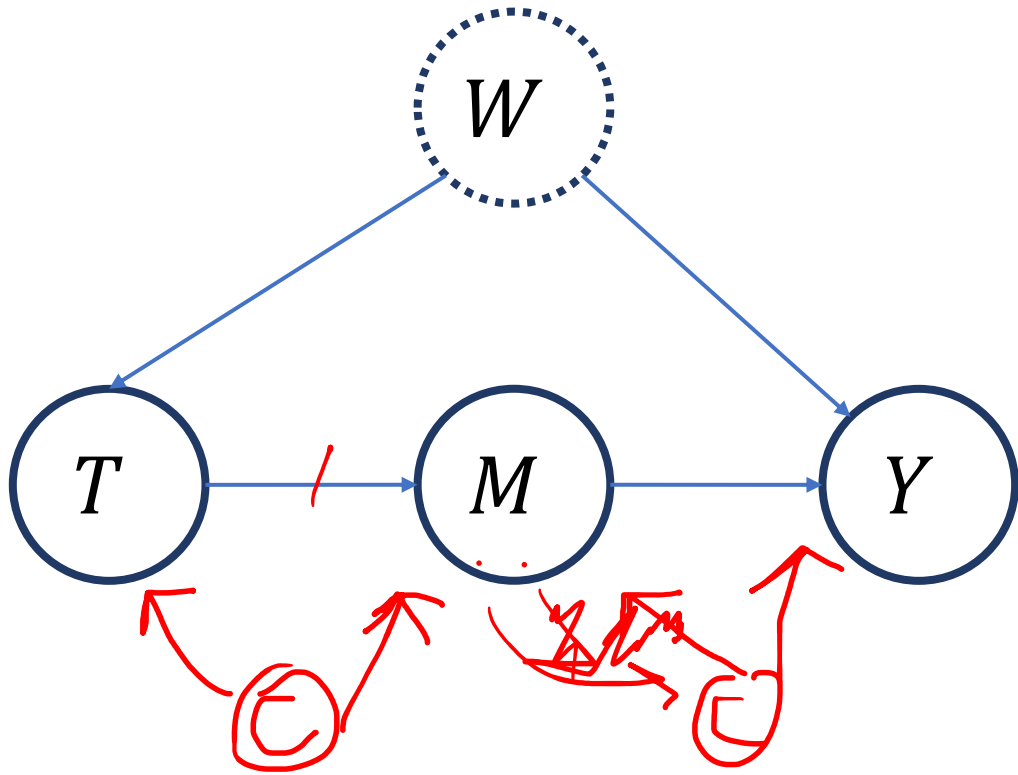
$$P(m|do(t)) = P(m|t)$$

$$P(y|do(m)) = \sum_t P(y|m, t) P(t)$$

$$\begin{aligned} P(y|do(t)) &= P(y|do(m))P(m|do(t)) \\ &= \sum_m P(m|t) \sum_{t'} P(y|m, t')P(t') \end{aligned}$$



# Frontdoor Adjustment

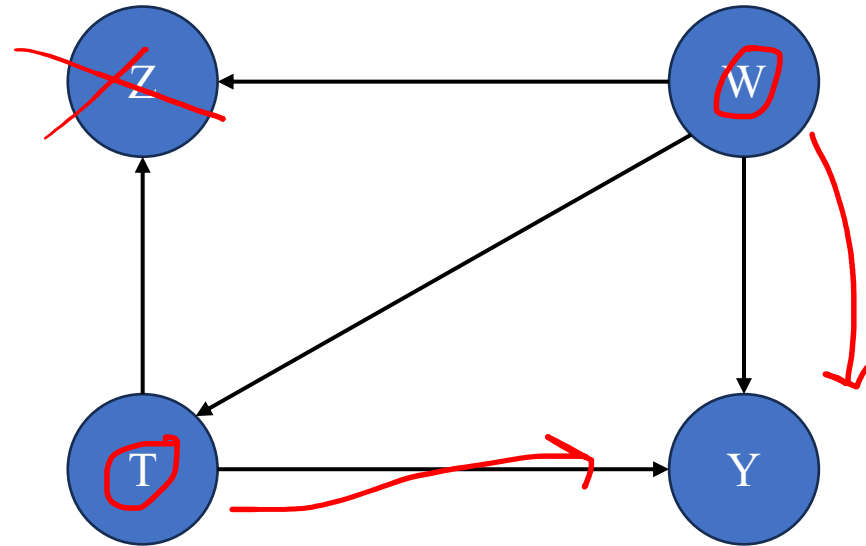


$M$  satisfies frontdoor criterion relative to  $T$  and  $Y$  if:

1. All causal paths from  $T$  to  $Y$  go through  $M$
2. No unblocked backdoor path from  $T$  to  $M$
3. All backdoor paths from  $M$  to  $Y$  are blocked by  $T$

# do-Calculus

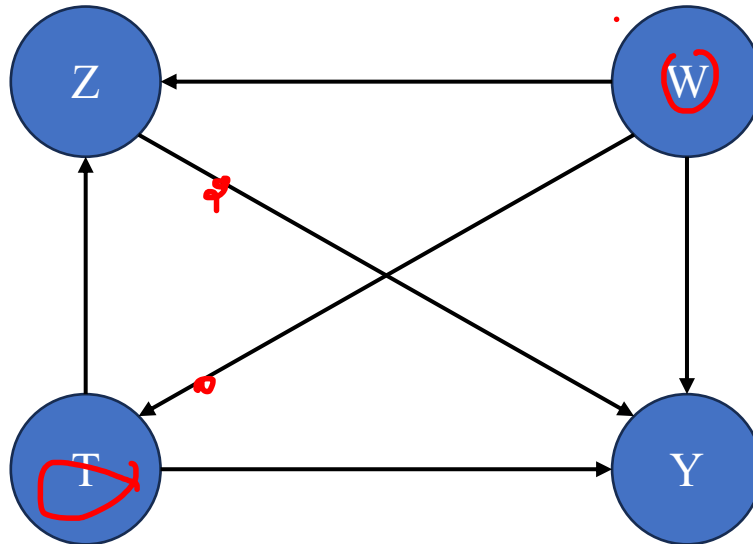
Rule 1:  $P(y|do(t), \underline{z}, w) = P(y|do(t), w)$  if  $(Y \perp\!\!\!\perp_{G_{\bar{T}}} Z | \underline{T}, W)$



# do-Calculus

Rule 1:  $P(y|do(t), z, w) = P(y|do(t), w)$  if  $Y \perp_{G_{\bar{T}}} Z | T, W$

Rule 2:  $P(y|do(t), \underline{do(z)}, w) = P(y|do(t), \underline{z}, w)$  if  $Y \perp_{G_{\bar{T}, \underline{Z}}} Z | T, W$

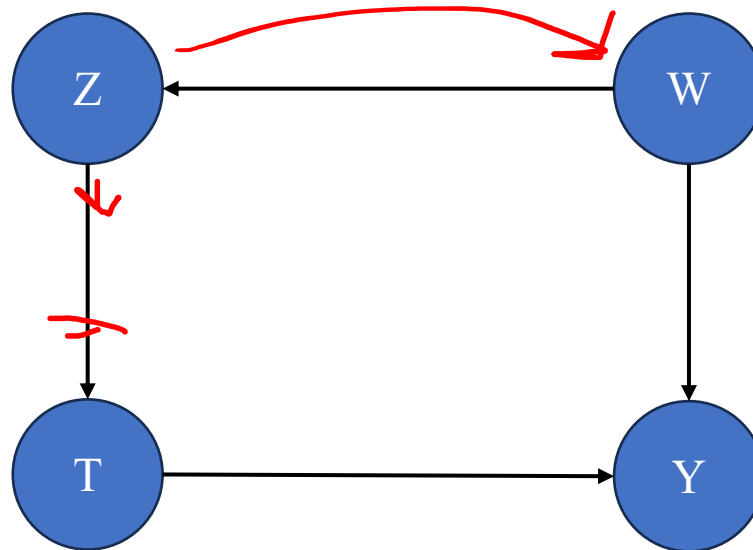


# do-Calculus

Rule 1:  $P(y|do(t), z, w) = P(y|do(t), w)$  if  $Y \perp_{G_{\bar{T}}} Z | T, W$

Rule 2:  $P(y|do(t), \underline{do(z)}, w) = P(y|do(t), \underline{z}, w)$  if  $Y \perp_{G_{\bar{T}, \underline{Z}}} Z | T, W$

Rule 3:  $P(y|do(t), \underline{do(z)}, w) = P(y|do(t), w)$  if  $Y \perp_{G_{\bar{T}, \underline{Z(W)}}} Z | T, W$



# Backdoor Adjustment

$W$  satisfies backdoor criterion relative to  $T$  and  $Y$  if:

1.  $W$  blocks all backdoor paths from  $T$  to  $Y$
2.  $W$  does not contain any descendants of  $T$

$$P(y|do(t)) = \sum_w P(y|t, w)P(w)$$