Potential Outcomes

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Overview

What are Potential Outcomes?

The Fundamental Problem of Causality & How to Get Around with It.

Key Assumptions

Randomized Control Trials

Warning: examples are heavily biased towards cats

Potential Outcomes

Estimating the effect of treatment/policy on the outcome of interest.

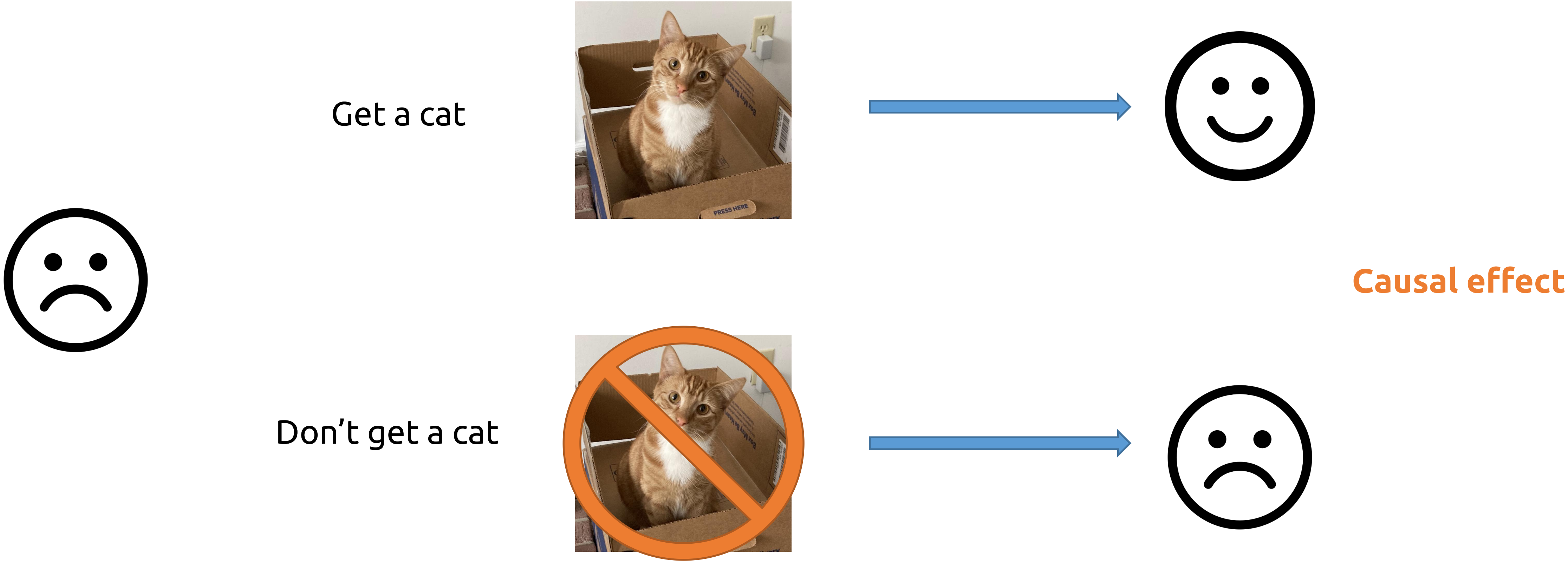
Get a cat

☺

☹

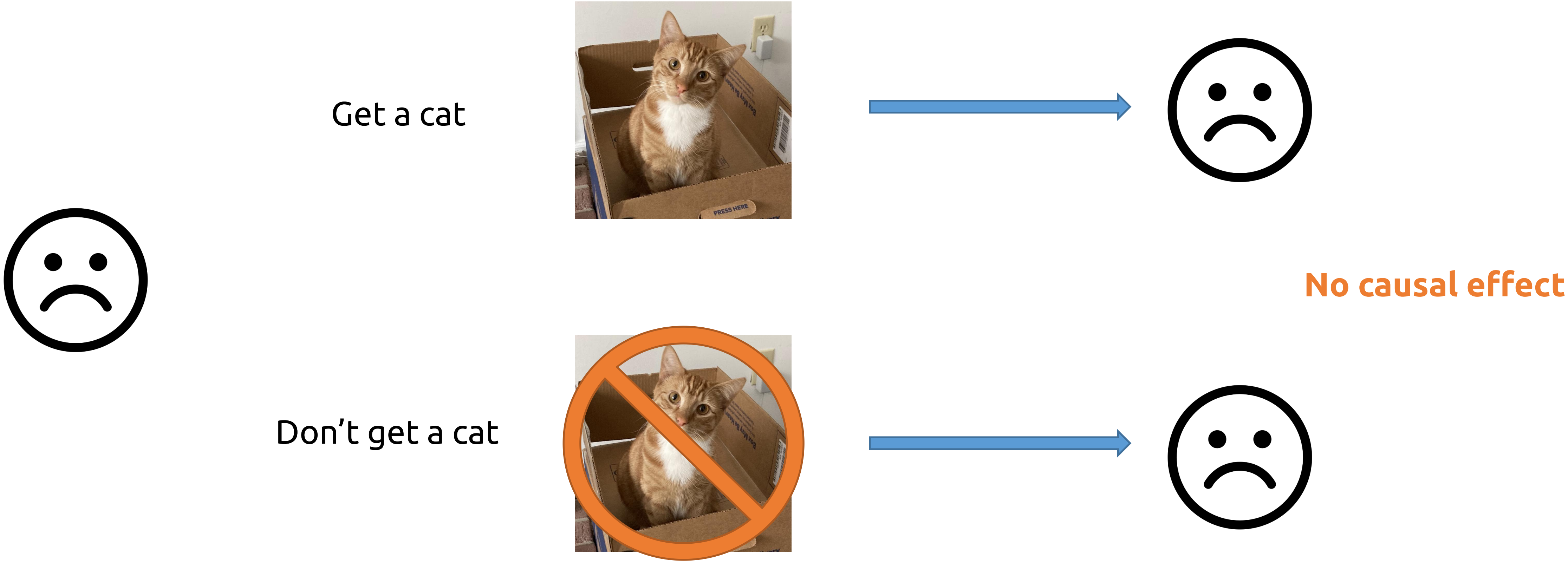
Potential Outcomes

Estimating the effect of treatment/policy on the outcome of interest.



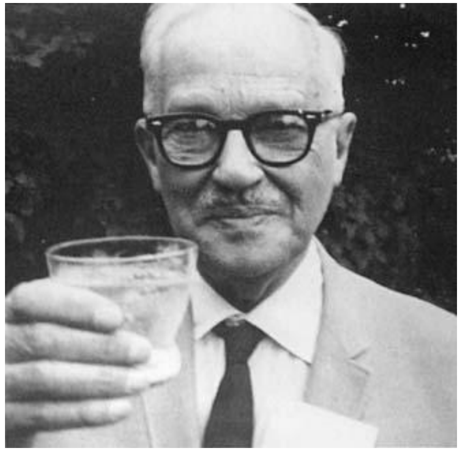
Potential Outcomes

Estimating the effect of treatment/policy on the outcome of interest.



Potential Outcomes Framework

Neyman-Rubin causal model



Jerzy Neyman

Donald Rubin

Does X cause Y?

If so, what's the magnitude of the effect?

Individual Treatment Effects (ITE)

$\mathrm{do}(T = 1)$

$Y_i(1) := Y_i \mid \mathrm{do}(T = 1)$

$\mathrm{do}(T = 0)$

$Y_i(0) := Y_i \mid \mathrm{do}(T = 0)$

Y_i: observed outcome

$Y_i(t)$: potential outcome

Individual Treatment Effects (ITE)

$\text{do}(T = 1)$

$Y_i(1) = 1$

$\text{do}(T = 0)$

$Y_i(0) = 0$

$$\text{ITE} = Y_i(1) - Y_i(0)$$
$$= 1 - 0 = 1$$

Fundamental Problem of Causal Inference

$\text{do}(T = 1)$

$Y_i(1) = 1$

Factual

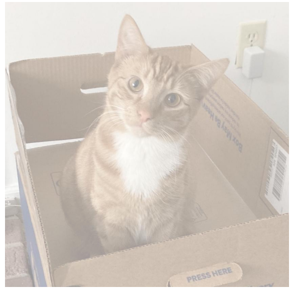
$\text{do}(T = 0)$

$Y_i(0) = 0$

Counterfactual

Fundamental Problem of Causal Inference

$\mathrm{do}(T = 1)$



$Y_i(1) = 1$

Counterfactual

$\mathrm{do}(T = 0)$

$Y_i(0) = 0$

Factual

We cannot observe $Y_i(1)$, $Y_i(0)$
at the same time

Fundamental Problem of Causal Inference

$\text{do}(T = 1)$

$Y_i(1) = 1$

Counterfactual

$\text{do}(T = 0)$

$Y_i(0) = 0$

Factual

Not a problem for ML models!
We can observe both in "simulations"

Fundamental Problem of Causal Inference

i	T	Y	$Y(1)$	$Y(0)$	$Y(1) - Y(0)$
0	0	0	?	0	?
1	1	1	1	?	?
2	0	0	?	0	?
3	0	0	?	0	?
4	1	0	0	?	?
5	1	1	1	?	?

Missing data problem

How to get around with it?

Holland, P. W. (1986). Statistics and Causal Inference.

Average Treatment Effect (ATE)

$$\mathbb{E}[Y(1) - Y(0)] = \mathbb{E}[Y|T=1] - \mathbb{E}[Y|T=0]$$

i	T	Y	$Y(1)$	$Y(0)$	$Y(1) - Y(0)$
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2	0	0	?	0	?
3	0	0	?	0	?
4	1	0	0	?	?
5	1	1	1	?	?

Average Treatment Effect (ATE)

$$\mathbb{E}[Y(1) - Y(0)] = \underline{\mathbb{E}[Y|T=1]} - \underline{\mathbb{E}[Y|T=0]}$$

i	T	Y	$Y(1)$	$Y(0)$	$Y(1) - Y(0)$
0	0	0		0	?
1	1	1	1		?
2	0	0		1	?
3	0	0		0	?
4	1	0	0		?
5	1	1	1		?

$$\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

Average Treatment Effect (ATE)

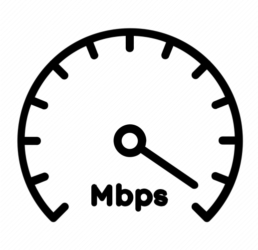
$$\underbrace{\mathbb{E}[Y(1) - Y(0)]}_{\text{Causal difference}} \neq \underbrace{\mathbb{E}[Y|T=1] - \mathbb{E}[Y|T=0]}_{\text{Associational difference}}$$

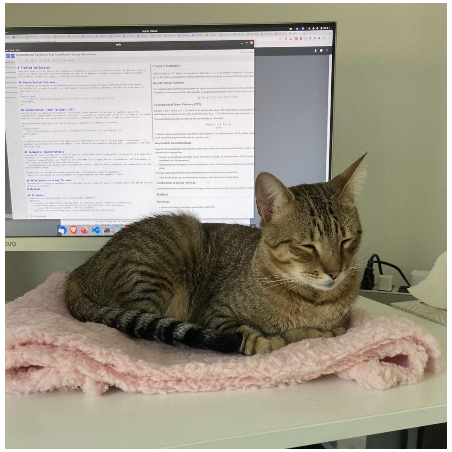
i	T	Y	$Y(1)$	$Y(0)$	$Y(1) - Y(0)$
0	0	0		0	?
1	1	1	1		?
2	0	0		1	?
3	0	0		0	?
4	1	0	0		?
5	1	1	1		?

Correlation ≠ Causation

Correlation ≠ Causation

Cat ownership is highly correlated with faster Internet speed





Correlation ≠ Causation

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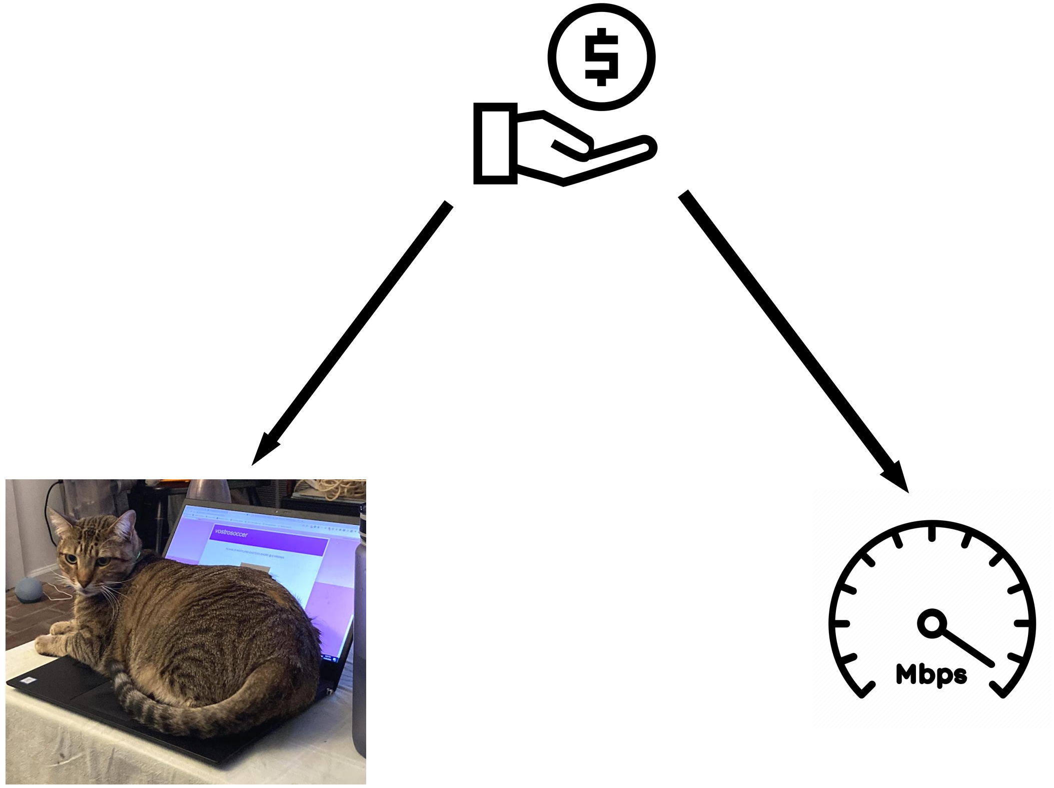
Common cause: Higher income



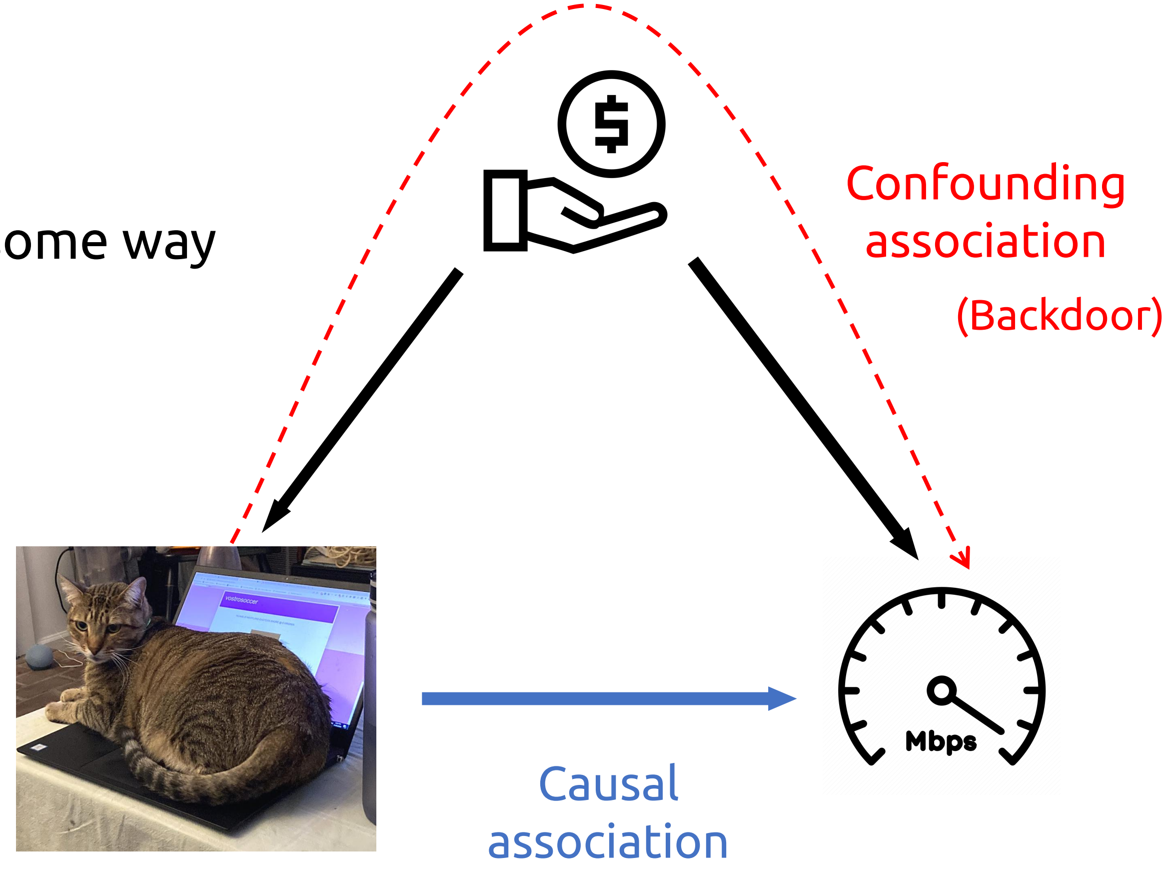
Correlation ≠ Causation

Cat ownership is highly correlated with faster Internet speed

Common cause: Higher income

→ Confounding

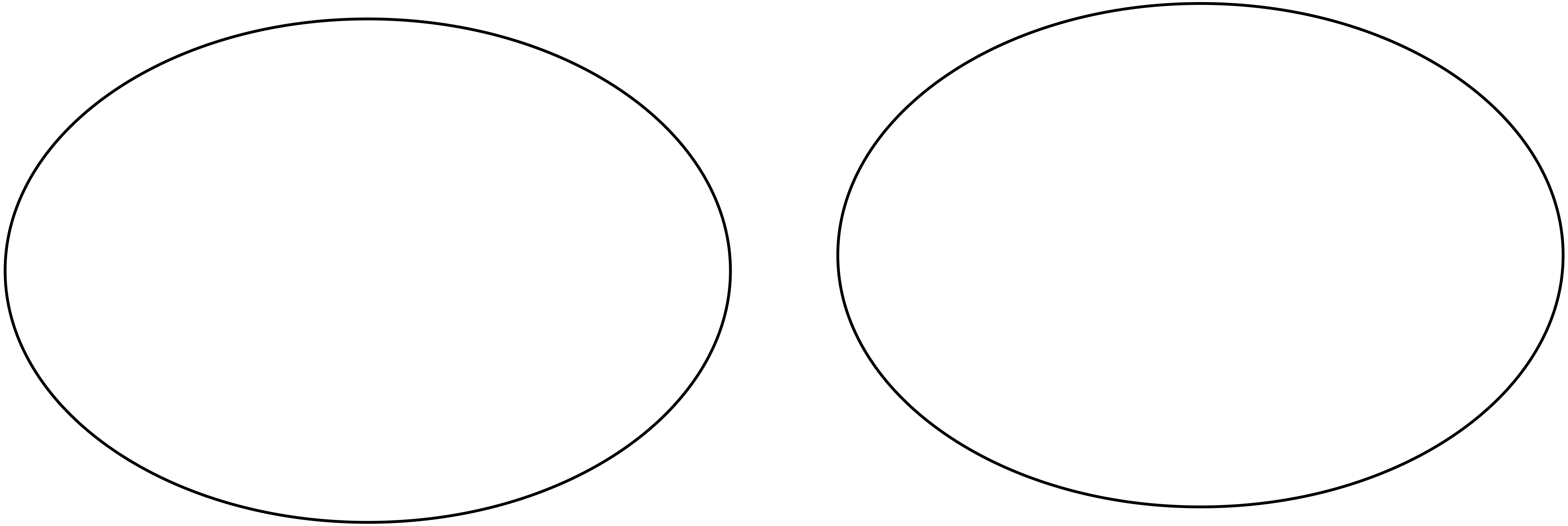
→ Cat owners differ from non-cat-owners in some way



Groups are incomparable

$$\mathbb{E}[Y(1) - Y(0)] \neq \mathbb{E}[Y|T=1] - \mathbb{E}[Y|T=0]$$

Own a cat ($T = 1$) Do not own a cat ($T = 0$)



Groups are incomparable

$$\mathbb{E}[Y(1) - Y(0)] \neq \mathbb{E}[Y|T = 1] - \mathbb{E}[Y|T = 0]$$

Own a cat ($T = 1$)

Do not own a cat ($T = 0$)

When Groups are comparable

$$\mathbb{E}[Y(1) - Y(0)] = \mathbb{E}[Y|T=1] - \mathbb{E}[Y|T=0]$$

Own a cat ($T = 1$)

Low income
High income
High income
Low income
Low income
High income
High income
Low income

Do not own a cat ($T = 0$)

High income
Low income
Low income
High income
Low income
High income
Low income
High income

ATE = Associational Difference

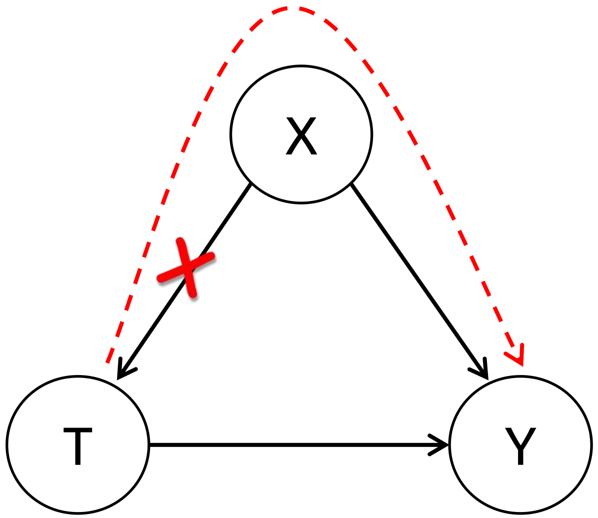
If the following assumptions are satisfied:

1. Ignorability / Exchangeability
2. Identifiability

Ignorability & Exchangeability $(Y(1), Y(0)) \perp\!\!\!\perp T$

Ignore how the treatment was assigned

Assume random assignment



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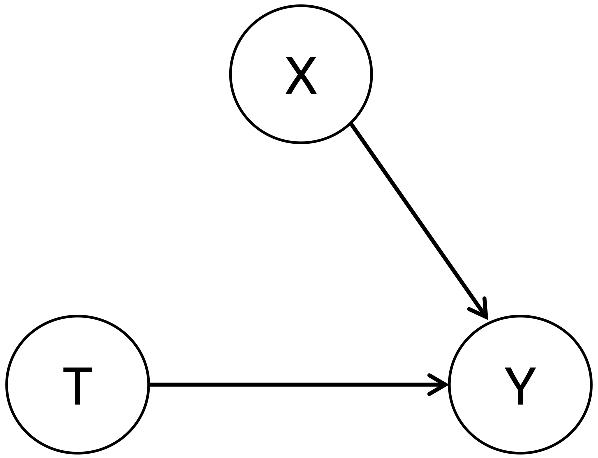

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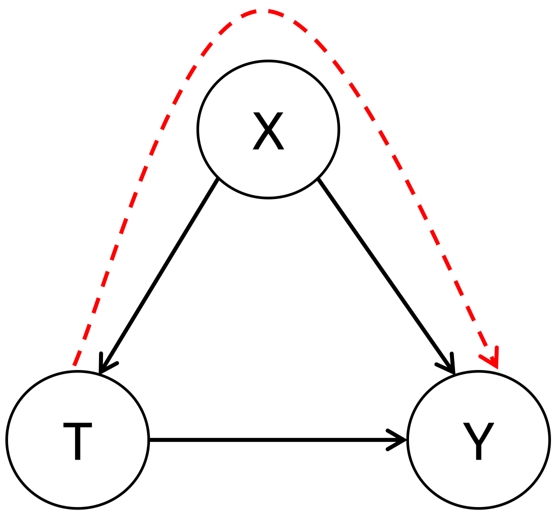
i	T	Y	$Y(1)$	$Y(0)$	$Y(1) - Y(0)$
0	0	0	?	0	?
1	1	1	1	?	?
2	0	0	?	0	?
3	0	0	?	0	?
4	1	0	0	?	?
5	1	1	1	?	?

Ignore the missing data problem



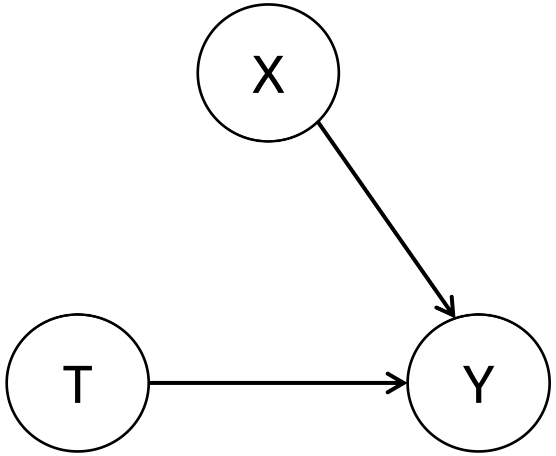
Ignorability & Exchangeability $(Y(1), Y(0)) \perp\!\!\!\perp T$

$$\mathbb{E}[Y(1) - Y(0)] = \mathbb{E}[Y(1)|T=1] - \mathbb{E}[Y(0)|T=0] \quad \text{(ignorability)}$$

$$= \mathbb{E}[Y|T=1] - \mathbb{E}[Y|T=0]$$

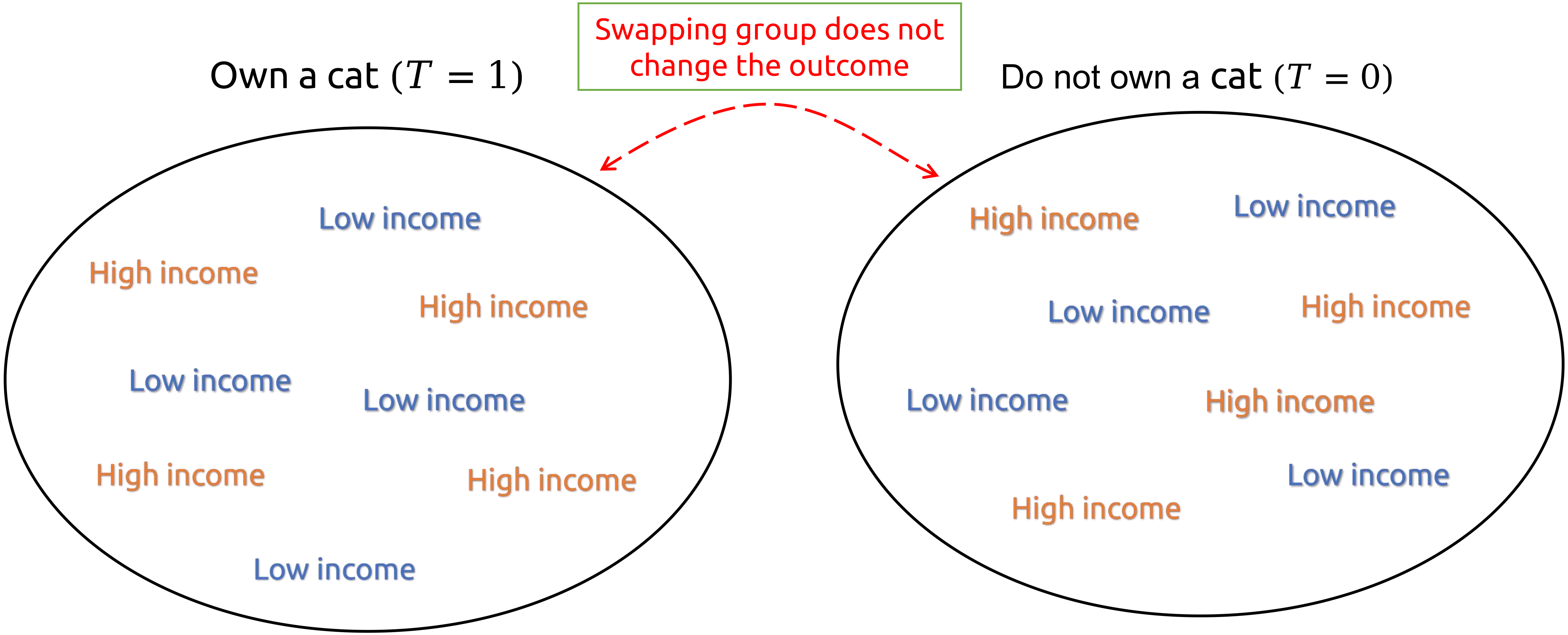
i	T	Y	$Y(1)$	$Y(0)$	$Y(1) - Y(0)$
0	0	0		0	?
1	1	1	1		?
2	0	0		0	?
3	0	0		0	?
4	1	0	0		?
5	1	1	1		?

Ignore the missing data



Exchangeability

$$\mathbb{E}[Y(1) - Y(0)] = \mathbb{E}[Y|T = 1] - \mathbb{E}[Y|T = 0]$$

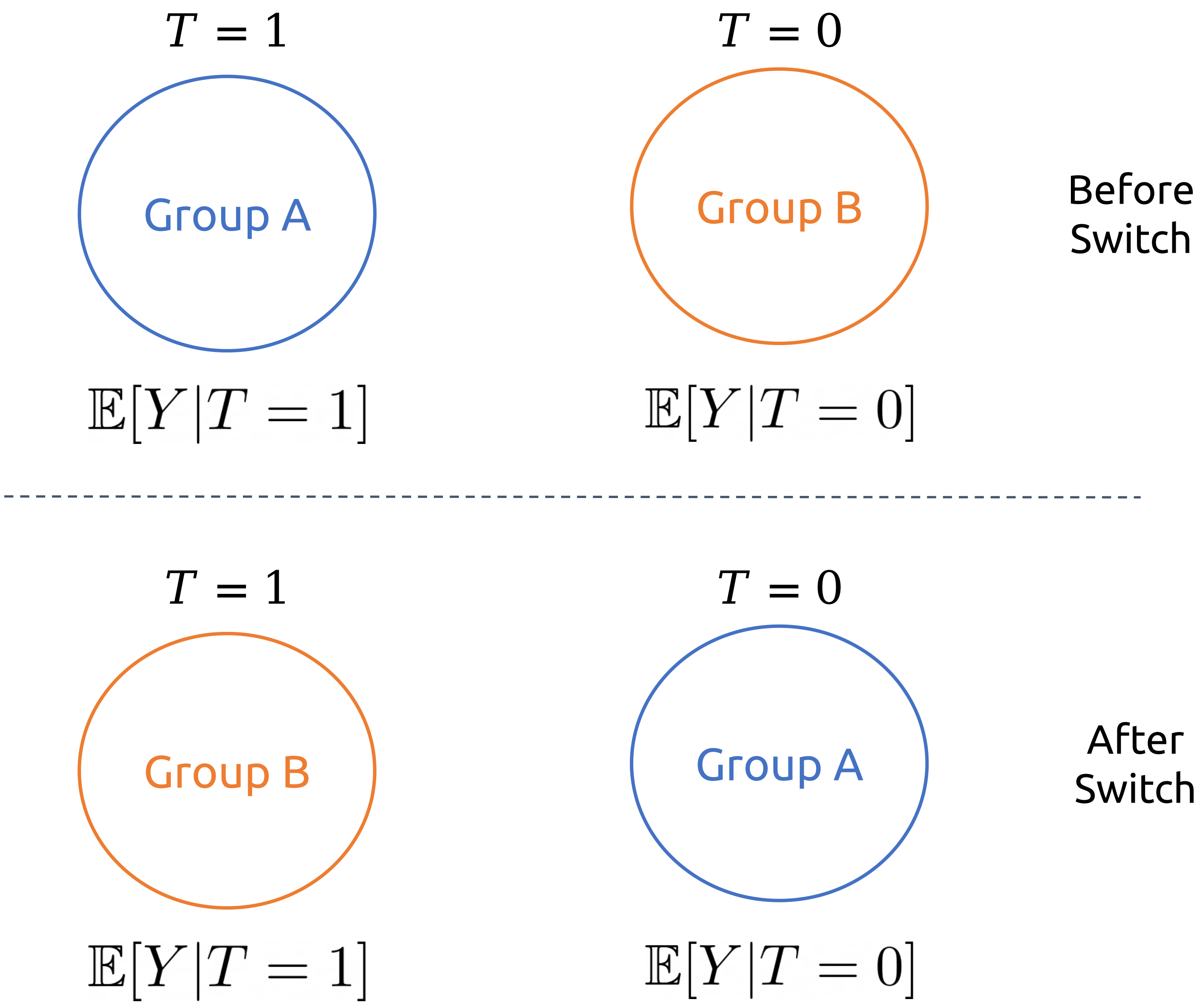


Exchangeability

$T = 1$ $\quad$ $T = 0$

$\mathbb{E}[Y|T = 1]$ $\quad$ $\mathbb{E}[Y|T = 0]$

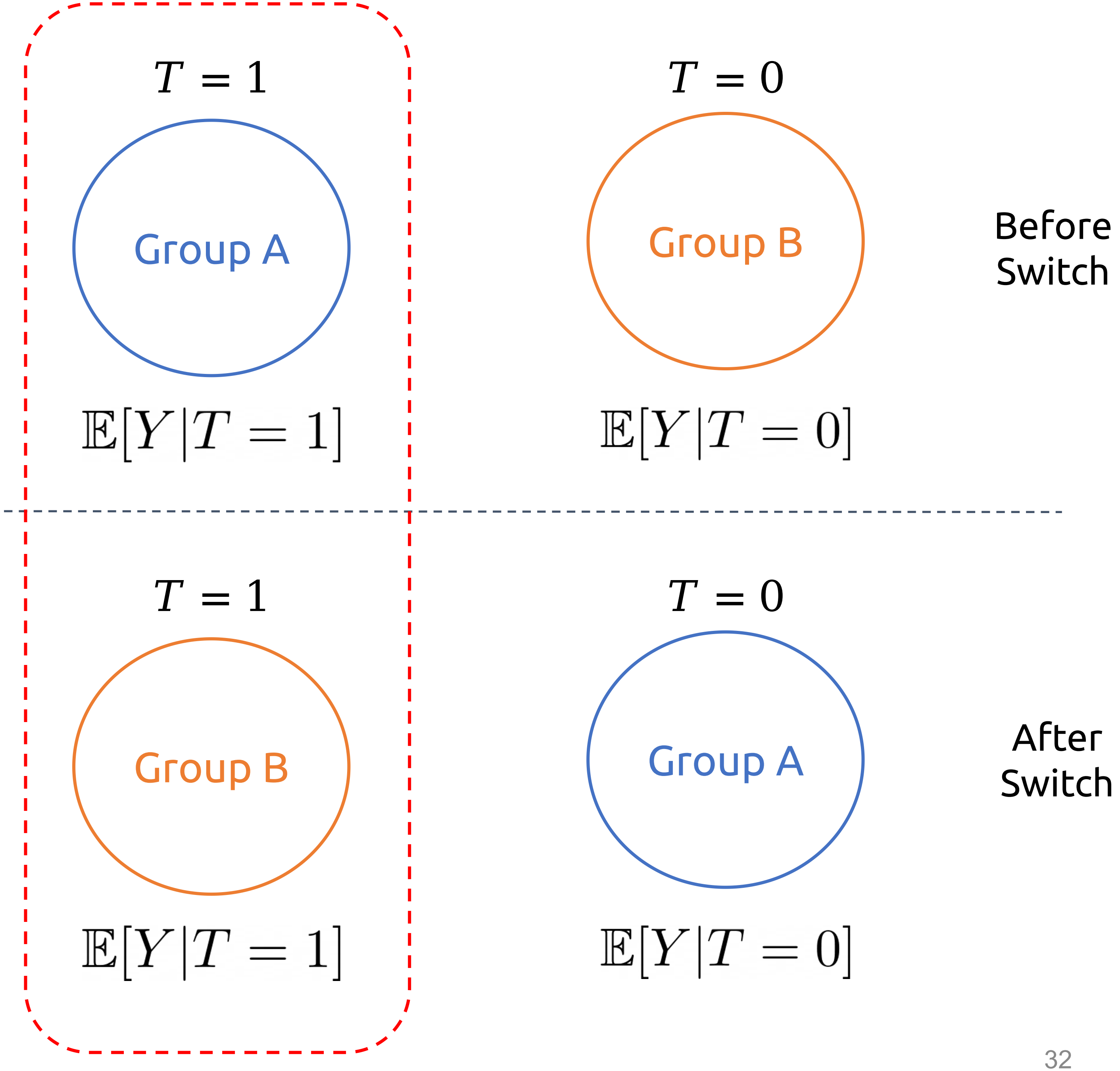
Exchangeability



Exchangeability

Before Switch After Switch

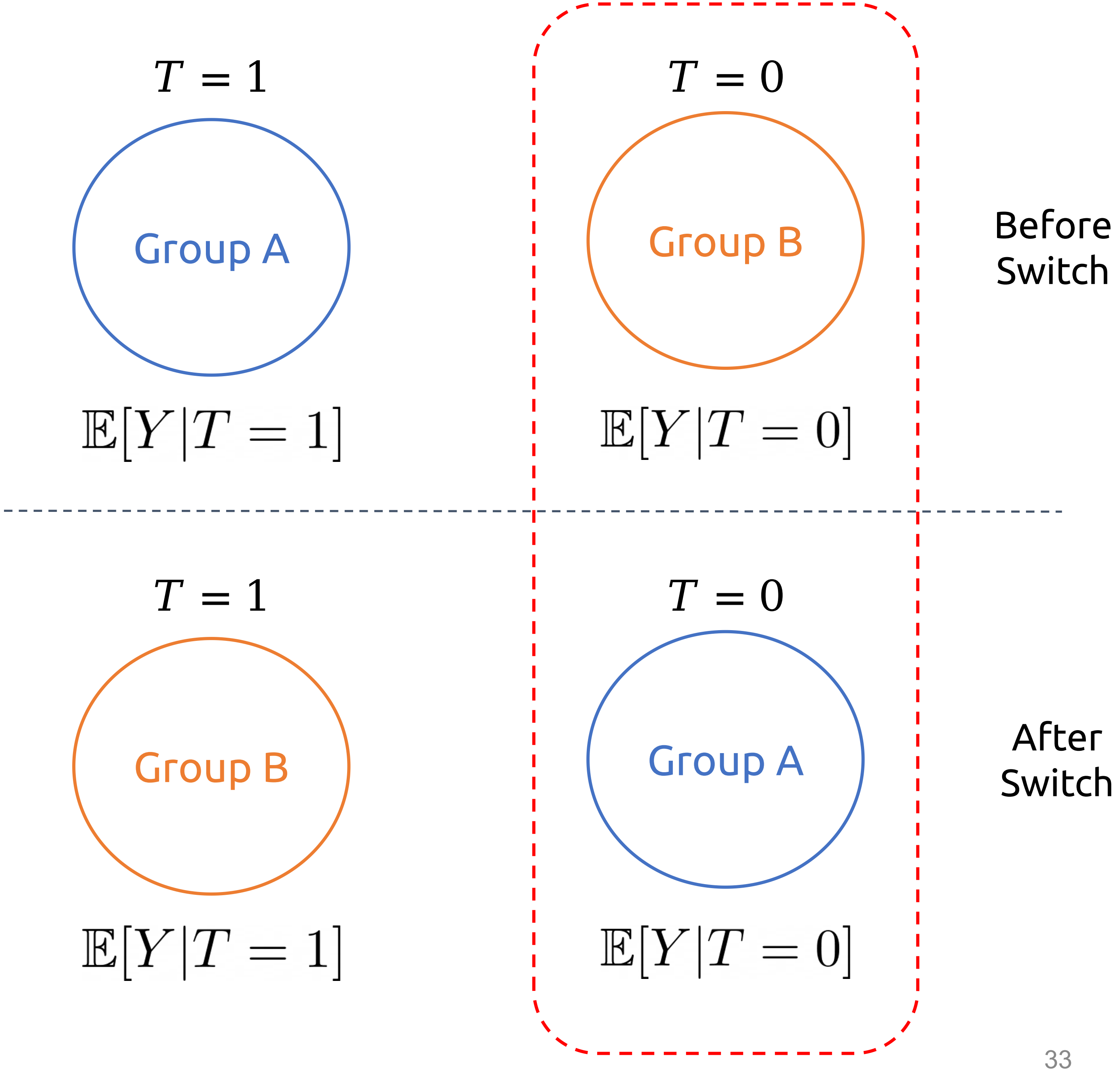
$$\mathbb{E}[Y(1)|T=1] = \mathbb{E}[Y(1)|T=0]$$



Exchangeability

Before Switch After Switch

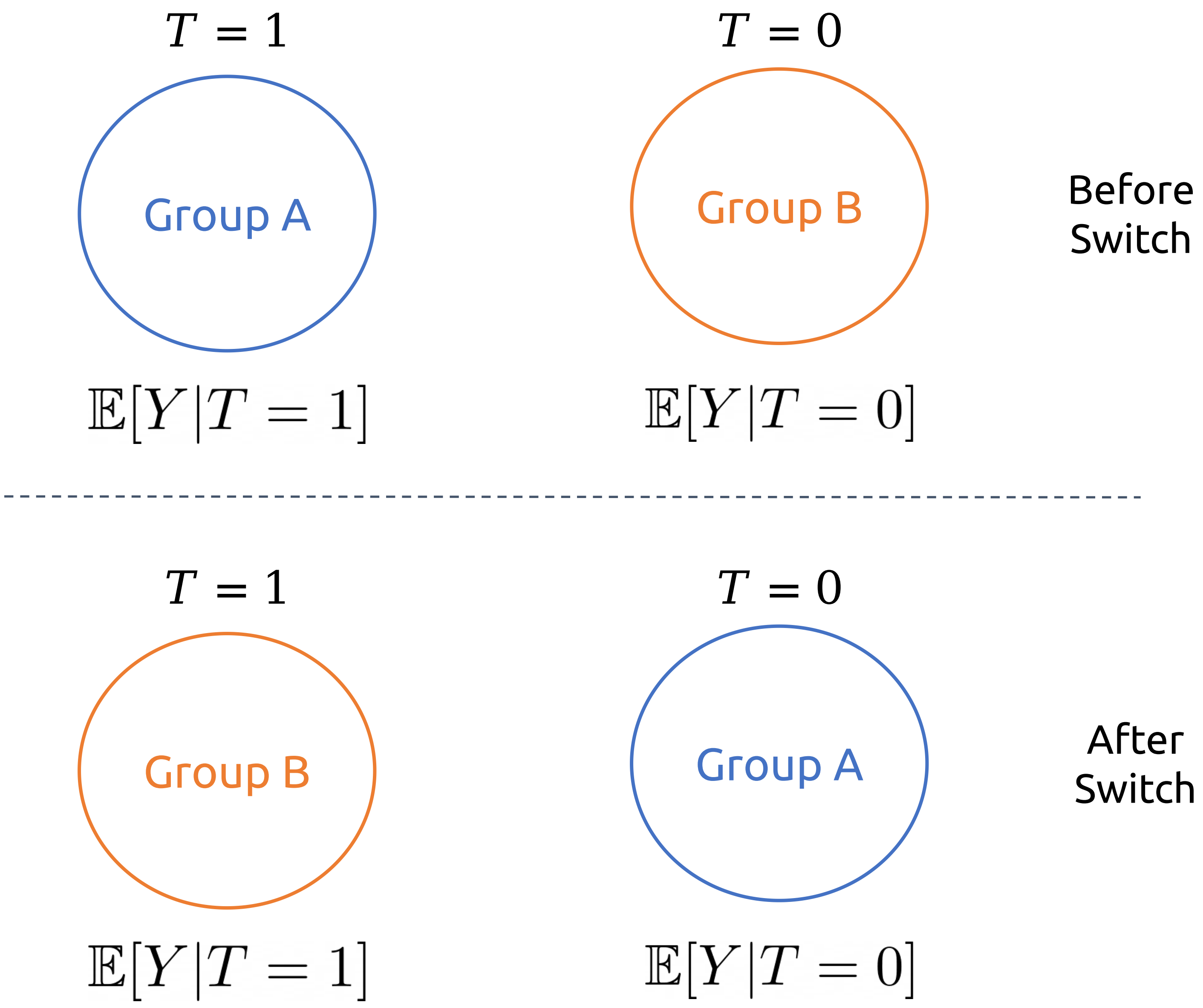
$$\mathbb{E}[Y(0)|T=0] = \mathbb{E}[Y(0)|T=1]$$



Exchangeability

$$\mathbb{E}[Y(1)|T = t] = \mathbb{E}[Y(1)]$$
$$\mathbb{E}[Y(0)|T = t] = \mathbb{E}[Y(0)]$$

=> $(Y(1), Y(0)) \perp\!\!\!\perp T$



Identifiability

A causal quantity is **identifiable** if we can compute from a purely statistical quantity.

$\mathbb{E}[Y(t)]$ $\mathbb{E}[Y|t]$

$$\mathbb{E}[Y(1) - Y(0)] = \mathbb{E}[Y(1)|T=1] - \mathbb{E}[Y(0)|T=0] \quad \text{(ignorability)}$$

Causal quantities

$$= \mathbb{E}[Y|T=1] - \mathbb{E}[Y|T=0]$$

Statistical quantities

Other Assumptions

Positivity: $0 < P(T = 1|X = x) < 1$

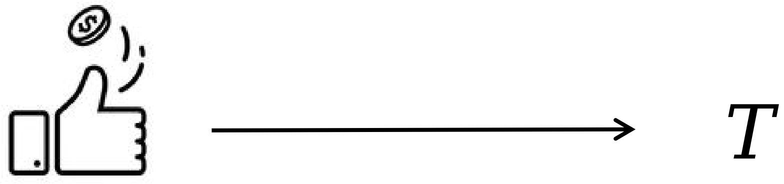
Stable unit-treatement value assumption (**SUTVA**)

→ No interference: outcome is unaffected by others' treatment

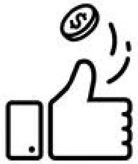
→ Consistency: If $T = t$, then $Y(t) = Y$

How do we achieve the assumptions realisticly?

Ans: Randomized control trial (RCT)



Randomized control trial (RCT)

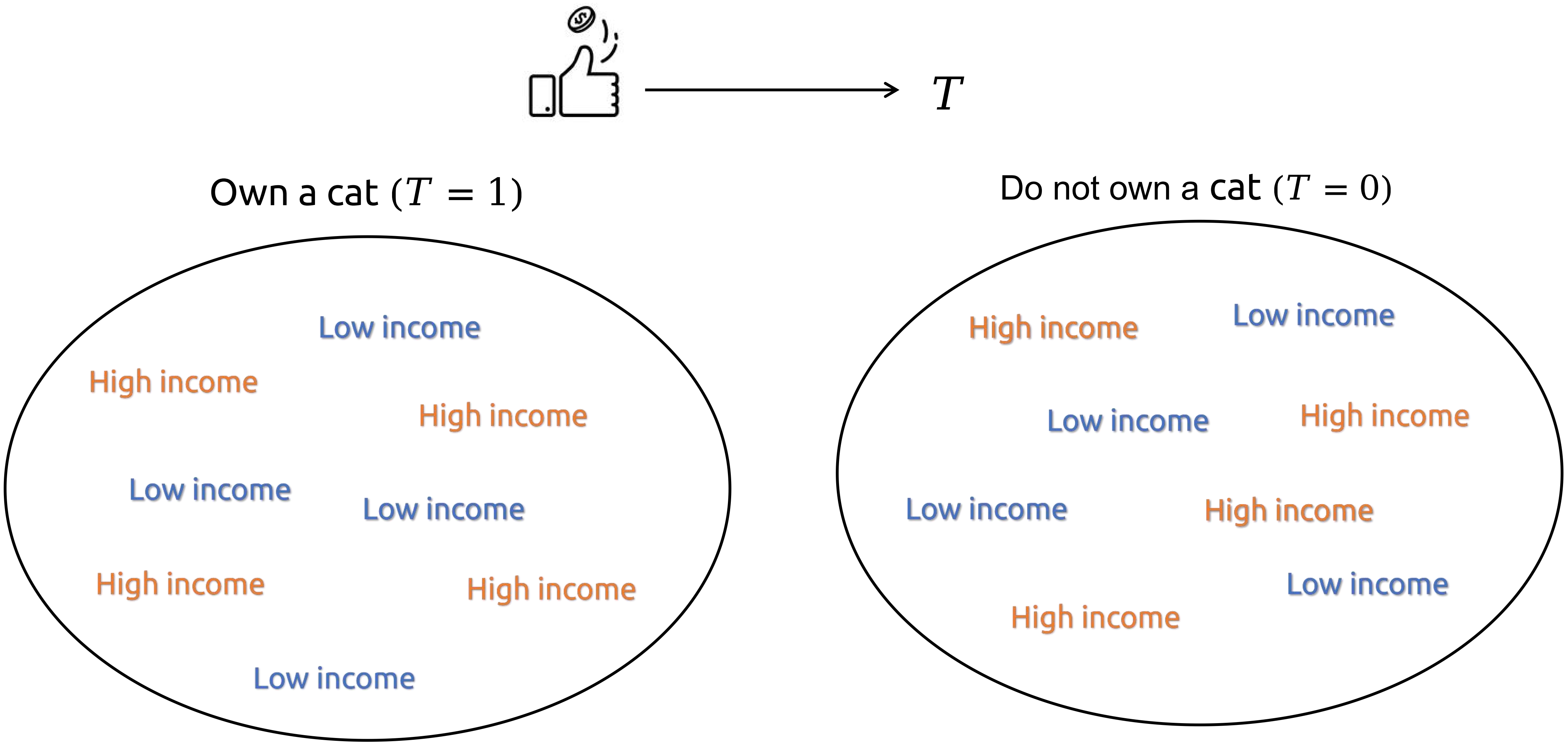


$\longrightarrow T$

Own a cat $(T = 1)$

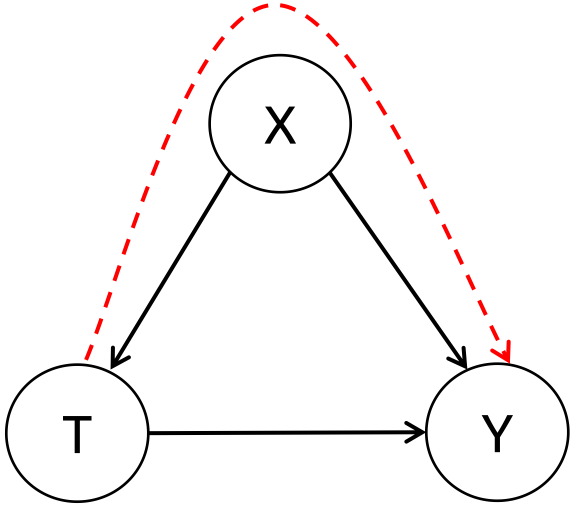
Do not own a cat $(T = 0)$

Randomized control trial (RCT)



Randomized control trial (RCT)

$$\mathbb{E}[Y(1) - Y(0)] \neq \mathbb{E}[Y|T=1] - \mathbb{E}[Y|T=0]$$



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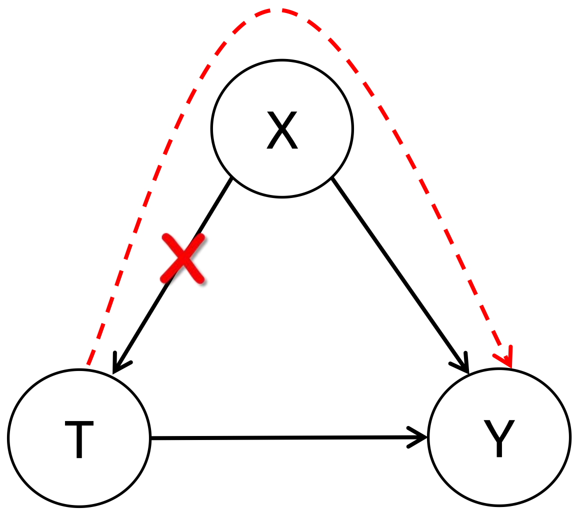

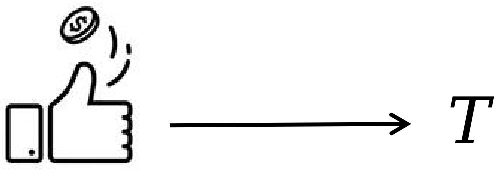

Randomized control trial (RCT)

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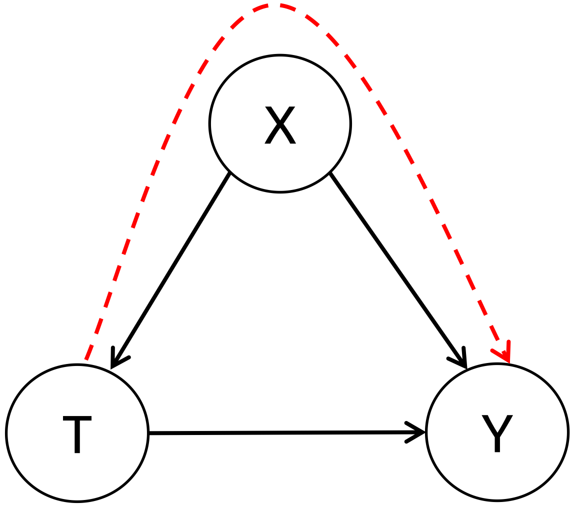
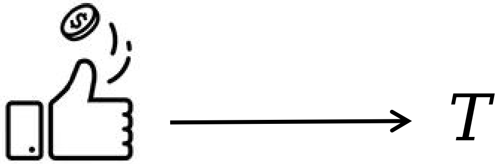
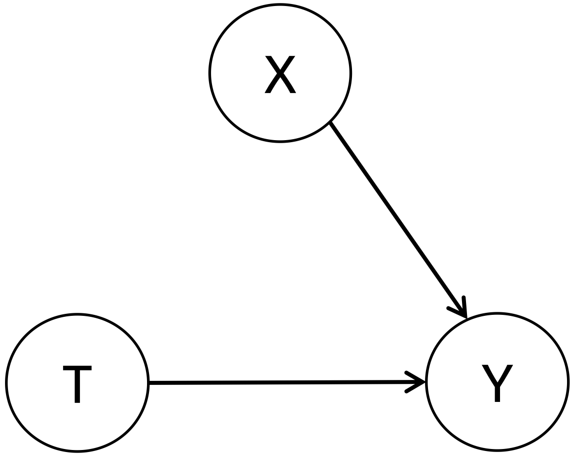






$$\mathbb{E}[Y(1) - Y(0)] = \mathbb{E}[Y|T=1] - \mathbb{E}[Y|T=0]$$

What if we can't do RCT?

We can't always randomize treatment

Ethical reasons: smoking → lung cancer

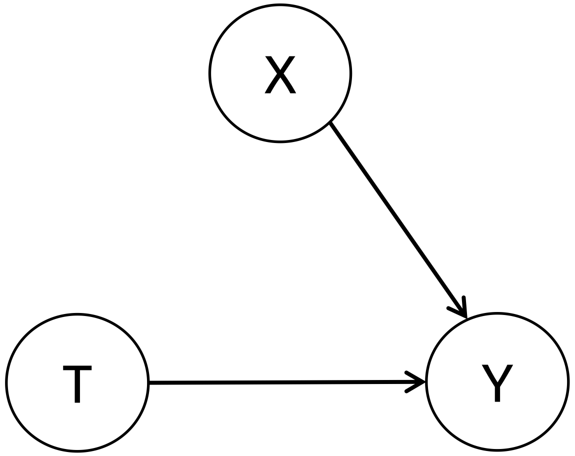
Infeasibility: gender → discrimination

Impossibility: a person's DNA → obesity

Active research: Causal inference in observational studies

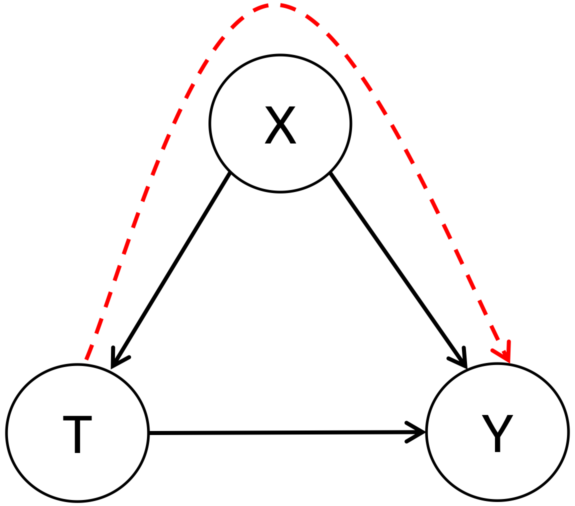
Conditional Exchangeability

Exchangeability



$$(Y(1), Y(0)) \perp\!\!\!\perp T$$

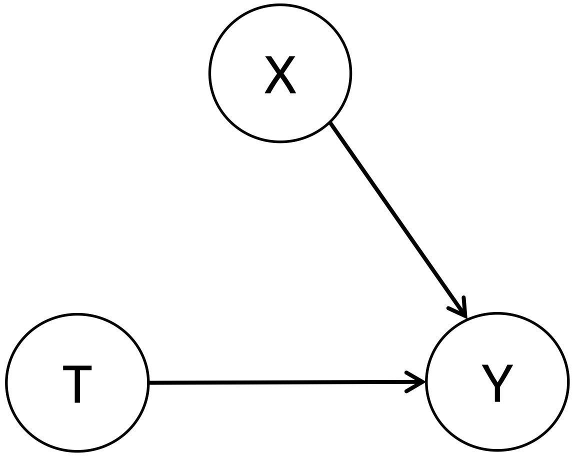
Conditional exchangeability



$$(Y(1), Y(0)) \perp\!\!\!\perp T \mid X$$

Conditional Exchangeability

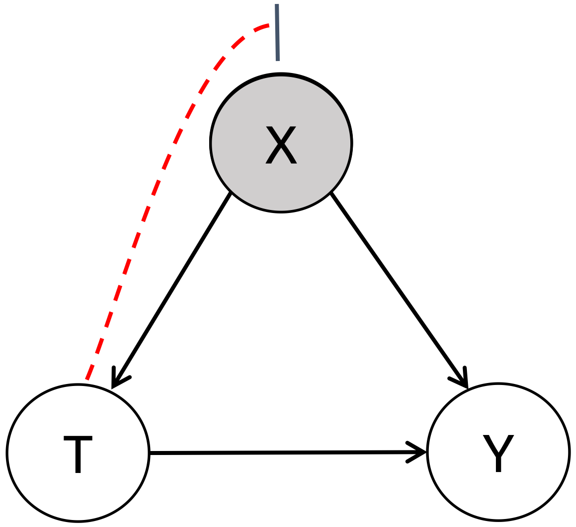
Exchangeability



$$(Y(1), Y(0)) \perp\!\!\!\perp T$$

(Unconfoundedness)

Conditional exchangeability



$$(Y(1), Y(0)) \perp\!\!\!\perp T \mid X$$

Conditional Average Treatment Effect (CATE)

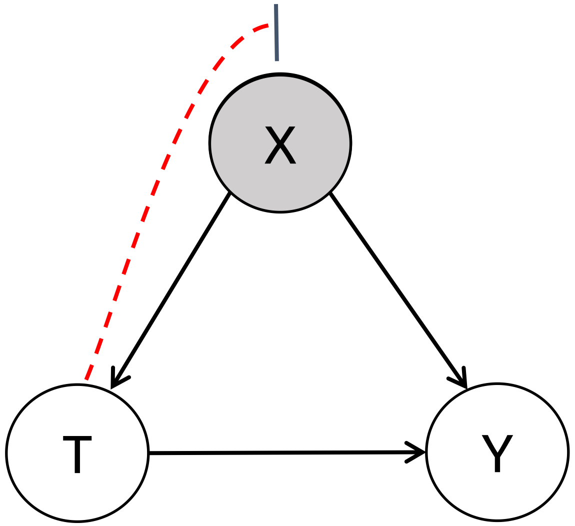
Conditional exchangeability $(Y(1), Y(0)) \perp\!\!\!\perp T \mid X$

Conditional ATE

$$\begin{aligned}\mathbb{E}[Y(1) - Y(0) \mid X] &= \mathbb{E}[Y(1) \mid X] - \mathbb{E}[Y(0) \mid X] \\ &= \mathbb{E}[Y(1) \mid T = 1, X] - \mathbb{E}[Y(0) \mid T = 0, X] \\ &= \mathbb{E}[Y \mid T = 1, X] - \mathbb{E}[Y \mid T = 0, X]\end{aligned}$$

Adjusting for Confounders

$$\mathbb{E}[Y(1) - Y(0)] = \mathbb{E}_X[\mathbb{E}[Y(1) \mid X] - \mathbb{E}[Y(0) \mid X]]$$
$$= \mathbb{E}_X[\mathbb{E}[Y \mid T = 1, X] - \mathbb{E}[Y \mid T = 0, X]]$$



Adjusting for Confounders

$$\mathbb{E}[Y(1) - Y(0)] = \mathbb{E}_X[\mathbb{E}[Y(1) \mid X] - \mathbb{E}[Y(0) \mid X]]$$
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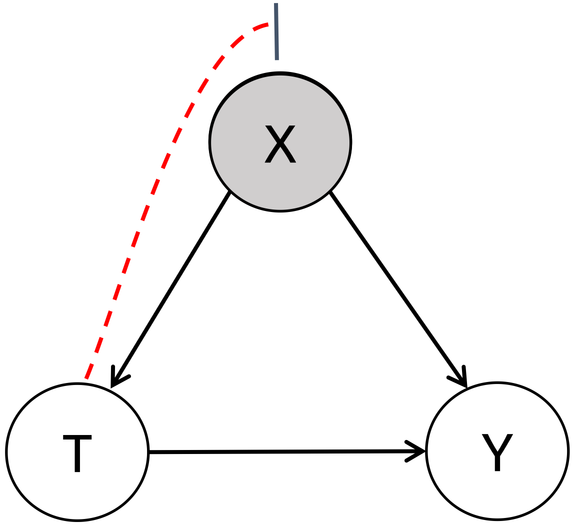


A set of variables W satisfy the **backdoor criterion** if:

1. W blocks all backdoor path from T to Y
2. W does not contain any descendants of T

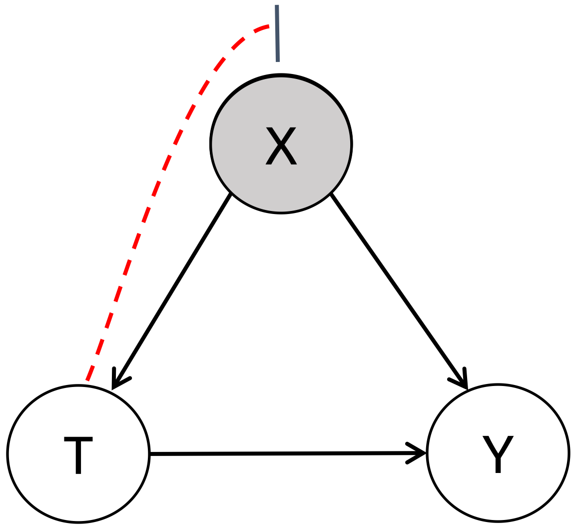
Backdoor adjustment

$$P(y \mid do(t)) = \sum_w P(y \mid t, w) P(w)$$

Unconfoundedness is Untestable

Conditional exchangeability

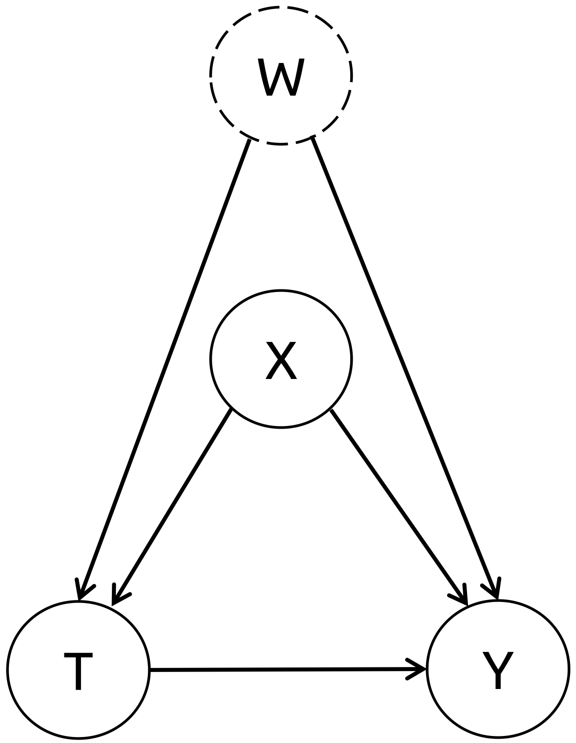
$$(Y(1), Y(0)) \perp\!\!\!\perp T \mid X$$



Unconfoundedness is Untestable

Conditional exchangeability

$$(Y(1), Y(0)) \not\perp\!\!\!\perp T \mid X$$



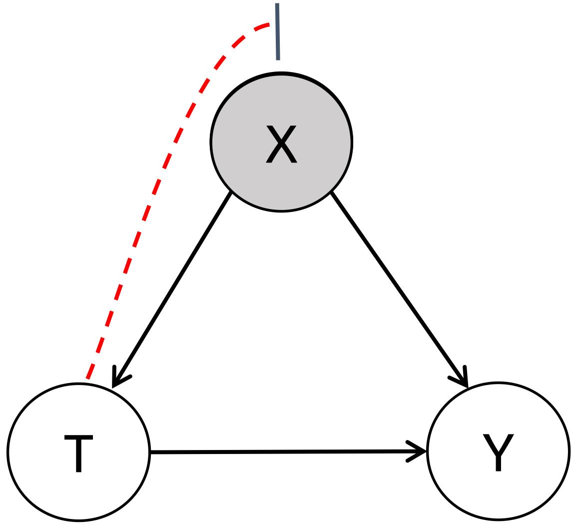
Summary

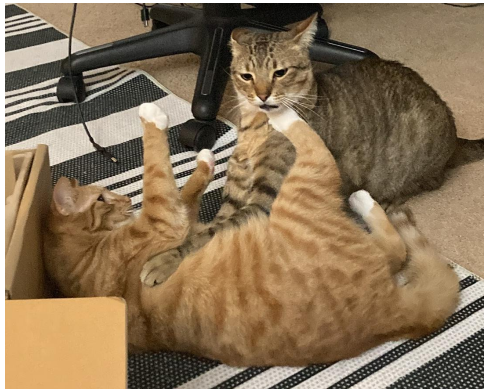
Potential outcomes

How to estimate causal effect

When association is causation (& when is not)

How to estimate causal effect with a confounder





Looking for Volunteers

Please fill out the form if you'd like to present (topic/paper)

Link can be found on Slack